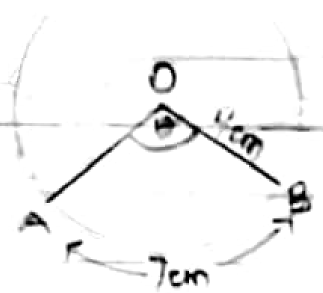
LEVEL 100, END-OF-SECOND SEMESTER EXAMINATION

TRIGONOMETRY

SOLUTION

Q1

a)



(i) Radian measure $= \dfrac{\text{Arc length}}{\text{Radius}} = \dfrac{7\text{cm}}{4\text{cm}} = 1.75^c$/Rad

(ii) Degree measure $= 1.75\pi^c \times \dfrac{180^\circ}{\pi} = 315^\circ$

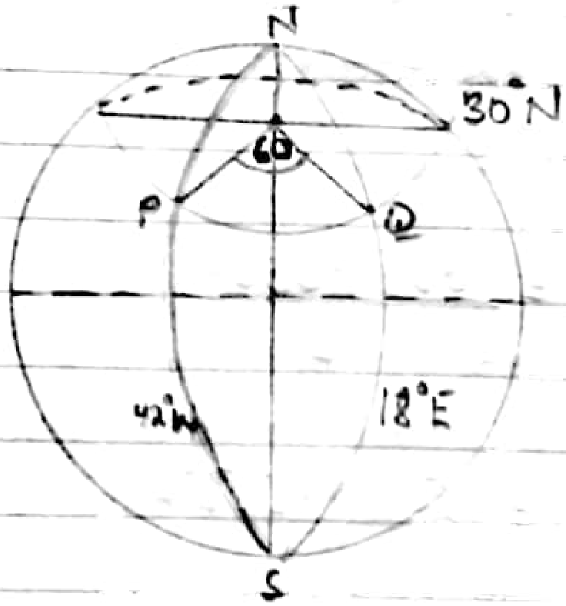
(iii) $\dfrac{\theta}{360^\circ} \times \pi r^2$, where $\theta = 315^\circ$, $r = 4$cm and $\pi = \dfrac{22}{7}$.

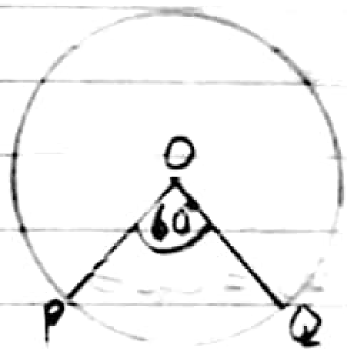
$\dfrac{315}{360} \times \dfrac{22}{7} \times (4)^2$

$\dfrac{315}{360} \times \dfrac{22}{7} \times 16$

$\dfrac{110{,}880}{2520} = 44\text{cm}^2$

1b)





The distance between P and Q along latitude 30°N

$= \frac{\theta}{360°} \times 2\pi r$

where $r = R\cos\theta$ and $\theta = 30°$, $R = 6{,}400$

$r = 6{,}400 \times \cos 30°$

$r = 6{,}400 \times 0.8910$

$r = 5{,}702.4417$ cm.

Hence:

$= \frac{60}{360} \times 2 \times 3.142 \times 5{,}702.4417$

1b...

$$= \frac{1}{6} \times 2 \times 3.142 \times 5{,}702.4417.$$

$$\frac{250{,}907.4\cancel{3}48}{\cancel{42}} = \frac{35811.3338}{6}$$

$$= 5968.5556$$

$$= 5{,}968.6 \text{ (1 decimal place).}$$

Q2.

2a) $(\sec\theta + \tan\theta)(1 - \sin\theta) = \cos\theta$

$$\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1 - \sin\theta)$$

~~Expanding the brackets:~~

$1($

Q2.

a) $(\sec\theta + \tan\theta)(1-\sin\theta) = \cos\theta.$

$$\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1-\sin\theta).$$

$$\left(\frac{1+\sin\theta}{\cos\theta}\right)(1-\sin\theta).$$

$$1\left(\frac{1+\sin\theta}{\cos\theta}\right) - \sin\theta\left(\frac{1+\sin\theta}{\cos\theta}\right)$$

$$\frac{1+\sin\theta}{\cos\theta} - \frac{\sin\theta - \sin^2\theta}{\cos\theta}.$$

$$\frac{1+\cancel{\sin\theta} - \cancel{\sin\theta} - \sin^2\theta}{\cos\theta}$$

$$\frac{1-\sin^2\theta}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta} = \frac{\cancel{\cos\theta}\cos\theta}{\cancel{\cos\theta}} = \cos\theta.$$

Hence $(\sec\theta + \tan\theta)(1-\sin\theta) = \cos\theta.$

2b) i) $3\cos\theta + 4\sin\theta = R\cos(\theta - a)$.

$R\cos(\theta - a)$.

$R(\cos\theta\cos a + \sin\theta\sin a)$.

$R\cos\theta\cos a + R\sin\theta\sin a$.

Dividing through by $\cos\theta$ and $\sin\theta$ respectively.

$$\frac{R\cancel{\cos\theta}\cos a}{\cancel{\cos\theta}} + \frac{R\cancel{\sin\theta}\sin a}{\cancel{\sin\theta}}$$

$R\cos a + R\sin a$.

So: $R\cos a + R\sin a = 3\cos\theta + 4\sin\theta$.

$R\cos a = 3$ ——— (1)

~~Rcos~~

$R\sin a = 4$ ——— (2)

Adding the squares of both eqns.

$R^2\cos^2 a + R^2\sin^2 a = 3^2 + 4^2$.

$R^2(\cos^2 a + \sin^2 a) = 9 + 16$

But $\cos^2 a + \sin^2 a = 1$

So: $R^2 = 25$

$\sqrt{R^2} = \sqrt{25}$

$R = 5$

ii) $R\sin a = 4$ ——— (2)

$R\cos a = 3$ ——— (1)

2b.)..

eqn ② ÷ eqn ①

$$\frac{R\sin a}{R\cos a} = \frac{4}{3}$$

$$\tan a = \frac{4}{3}$$

$$a = \tan^{-1}(4/3)$$

$$a = \cancel{59.03}\ 53.13^\circ$$

~~Showing~~

Hence: $3\cos\theta + 4\sin\theta = R\cos(\theta - a)$.

$3\cos\theta + 4\sin\theta = 5\cos(\theta - 53.13^\circ)$.

Q3.(a) $y = 2\cos(3x - \pi)$

Amplitude $= 2$

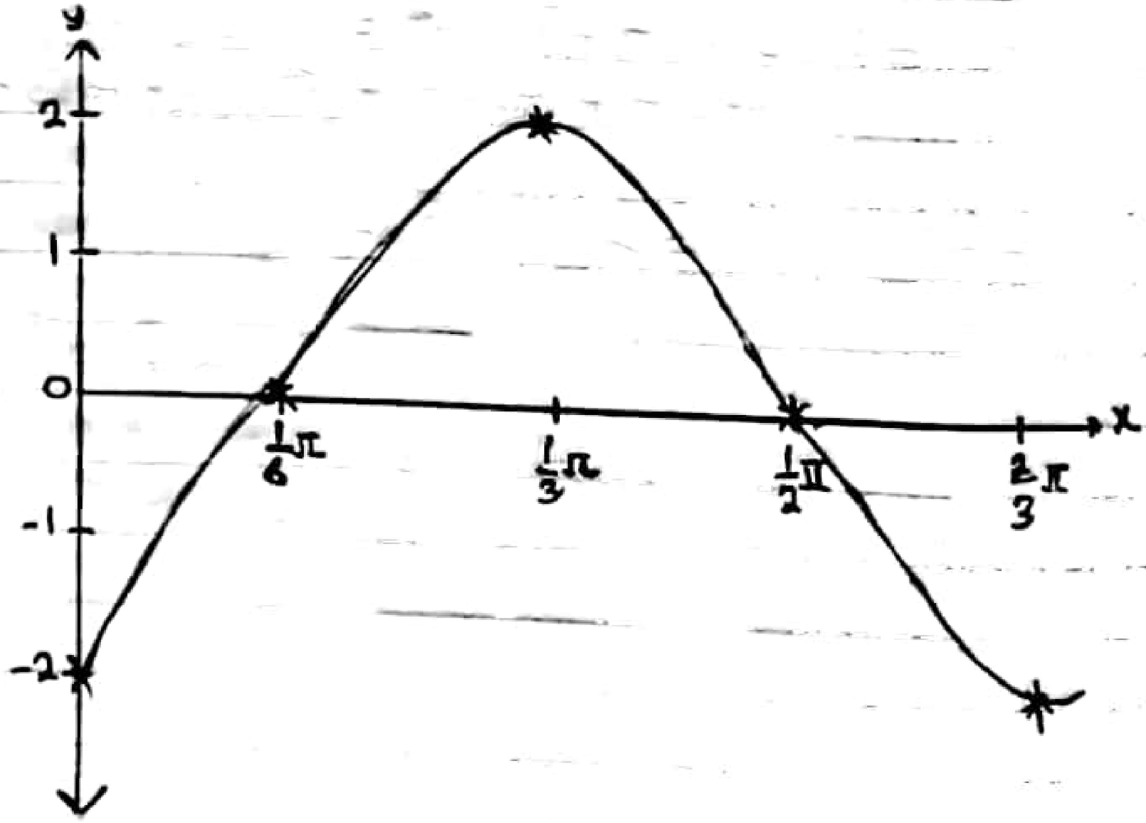
Period $= \dfrac{2\pi}{B}$, where $B = 3$.

$= \dfrac{2}{3}\pi$

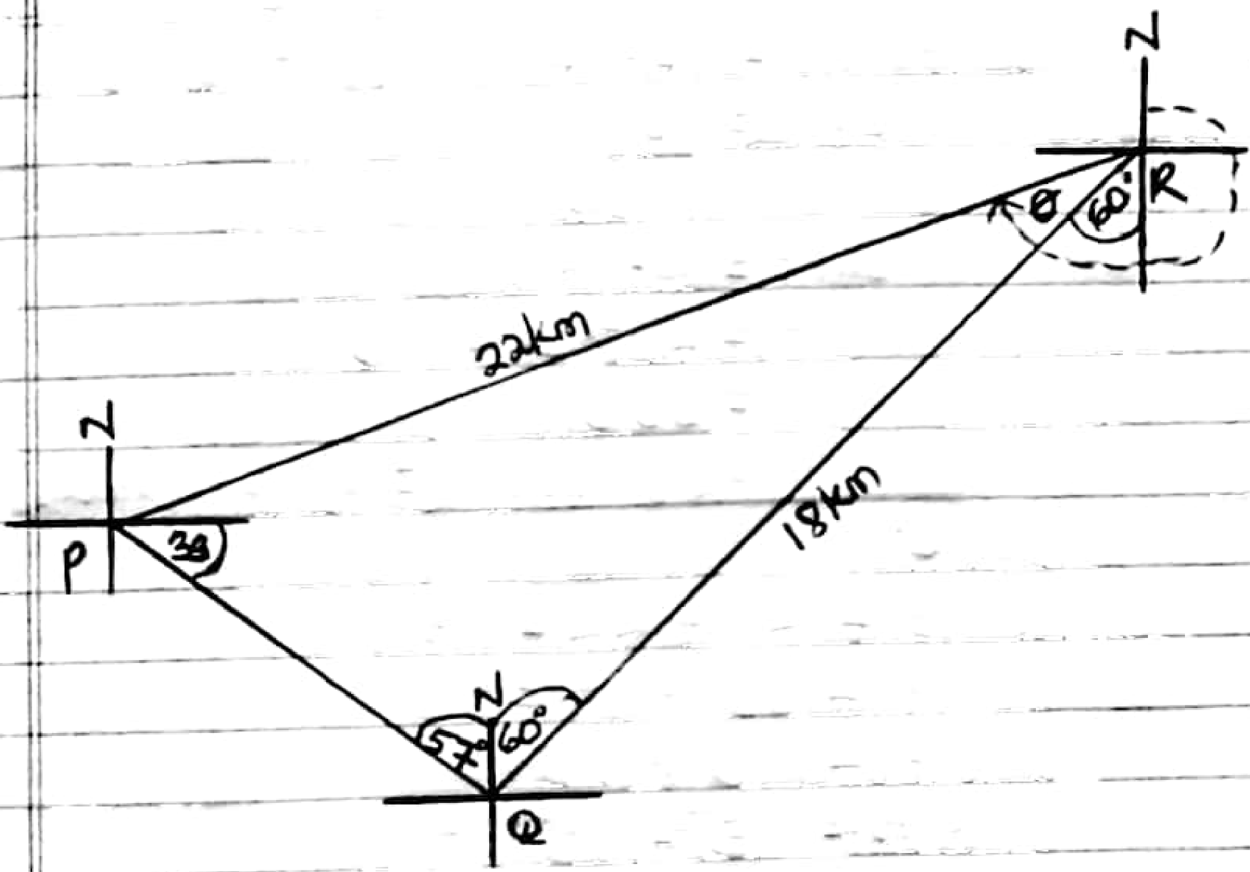
Phase shift $= 3x - \pi = 0$

$3x = \pi$

$x = \dfrac{\pi}{3}$.

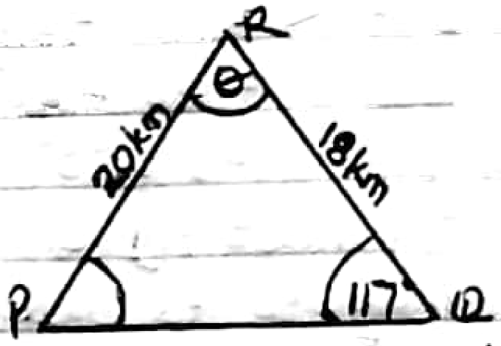


(Q3b).



(i) $\angle PQR = 57° + 60° = 117°$

(ii)



Using the sine rule:

$$\frac{18}{\sin P} = \frac{20}{\sin 117°}$$

$$20 \sin P = 18 \times \sin 117°$$

Q3b...

$20 \sin P = 16.0381$

$\sin P = \dfrac{16.0381}{20}$

$\sin P = 0.8019$

$P = \sin^{-1}(0.8019)$

$P = 53.31^\circ$

$P = 53^\circ$ (Nearest whole number).

Hence:

$\theta + \angle P + \angle Q = 180^\circ$

$\theta + 53^\circ + 117^\circ = 180^\circ$

$\theta + 170^\circ = 180^\circ$

$\theta = 180^\circ - 170^\circ$

$\theta = 10^\circ$

The bearing of P from R $= 180^\circ + 60^\circ + \theta$

$= 180^\circ + 60^\circ + 10^\circ$

$= \cancel{\$}\ 250^\circ$

Therefore the helicopter must fly on a bearing of 250° to return directly to its base.

Q4(a) $\cos^4 x = (\cos^2 x)^2$

$$\cos^4 x = \left(\frac{1}{2}(1+\cos 2x)\right)^2$$

$$\cos^4 x = \frac{1}{4}(1 + 2\cos 2x + \cos^2 2x).$$

But $\cos^2 2x = \frac{1}{2}(1+\cos 4x)$

$$\cos^4 x = \frac{1}{4}\left(1 + 2\cos 2x + \frac{1}{2}(1+\cos 4x)\right)$$

$$\cos^4 x = \frac{1}{4}\left(1 + 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x\right)$$

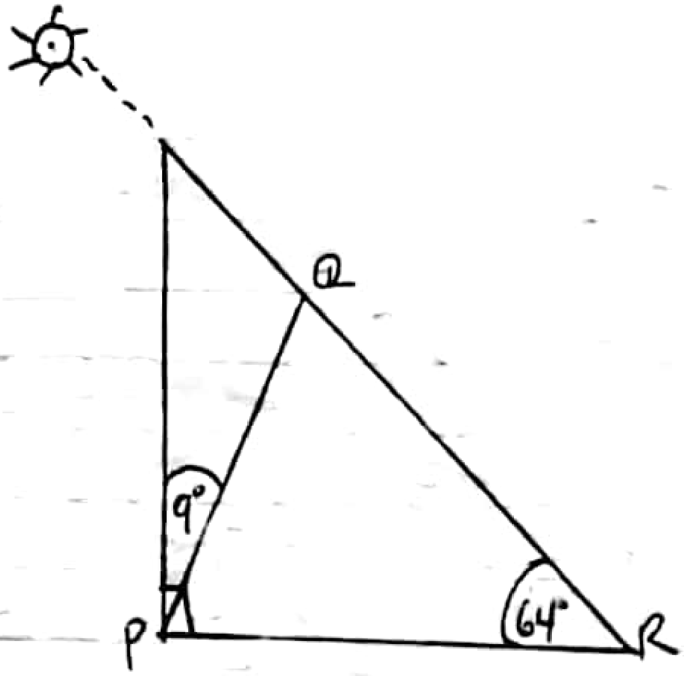
$$\cos^4 x = \frac{1}{4}\left(\frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x\right).$$

By factorizing $\frac{1}{2}$ out of the expression in the bracket:

$$\cos^4 x = \frac{1}{4} \times \frac{1}{2}\left(3 + 4\cos 2x + \cos 4x\right)$$

$$\cos^4 x = \frac{1}{8}\left(3 + 4\cos 2x + \cos 4x\right)$$

(4b)



$\angle QPR = 90° - 9° = 81°$

$\angle PQR = 180° - (64° + 81°)$

$\angle PQR = 180° - 145°$

$\angle PQR = 35°$

Using the sine rule:

$$\frac{PQ}{\sin 64°} = \frac{21}{\sin 35}$$

Solved by:
APPIAH-KUBI PRINCE (PROF.)

$$PQ \sin 35 = 21 \times \sin 64.$$

$$PQ = \frac{21 \times \sin 64}{\sin 35} = \frac{18.8747}{0.5736}$$

$PQ = 32.9056.$

$PQ = 33$ feet.

Therefore the pole is approximately 33 feet.

THE END.